IOSR Journal of Engineering (IOSRJEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
Vol. 05, Issue 07 (July. 2015), ||V1|| PP 50-60

On $\phi-$Pseudo Symmetric Lorentzian $\beta-$Kenmotsu Manifolds

N. V. C. SHUKLA & RAM ASRAY VERMA*
Department of Mathematics & Astronomy, University of Lucknow
Lucknow, 226007, UP INDIA

[2010] Mathematics Subject Classification, 30C45, 30C50.
Keywords and phrases: Sasakian manifold, curvature tensor, Ricci tensor, kenmotsu manifolds, concircularly curvature tensor.

Abstract: - The object of the present paper is to study $\phi-$pseudo symmetric and $\phi-$pseudo Ricci symmetric Lorentzian $\beta-$kenmotsu manifold and we also studied $\phi-$pseudo concircularly symmetric Lorentzian $\beta-$kenmotsu manifold and obtaind some interesting results.

I. INTRODUCTION

In 1969 , Tanno [29] classified connected almost contact metric manifolds whose automorphism groups possess the maximum dimension. For such a manifold,the sectional curvature of plane sections containing ξ is a constant, say c .It is already proved that they could be divided into three classes: (i) The homogeneous normal contact Riemannian manifolds with $c>0$, (ii) The global Riemannian products of a line or a circle with a K$\ddot{a}$hler manifold of constant holomorphic sectional curvature if $c=0$, and (iii) The warped product space RX_fC^n if $c<0$. It is known that the manifolds of class (i) are characterized by admitting a Sasakian structure. The manifolds of class (ii) are characterized by a tensorial relation admitting a cosymplectic structure. Kenmotsu [19] characterized the differential geometric properties of the manifolds of class (iii), which are now a days called Kenmotsu manifolds and later studied by De [10], Shaikh [25] , praksha [24] and others.

As a generalization of both Sasakian and Kenmotsu manifolds, Oubina introduced the notion of trans-Sasakian manifolds, which are closely related to the locally conformal K$\ddot{a}$hler manifolds. A trans-Sasakian manifolds of type $(0,0)$, $(\alpha,0)$ and $(0,\beta)$ are called the cosymplectic, $\alpha-$Sasakian and $\beta-$Kenmotsu manifolds respectively, α,β being scalar functions. In particular, if $\alpha=0$, $\beta=1$; and $\alpha=1,\beta=0$ then, a trans-Sasakian manifold will be a Kenmotsu and Sasakian manifold respectively.

The study of Riemann symmetric manifolds began with the work of Cartan [3]. A Riemannian manifold (M^n,g) is said to be locally symmetric due to Cartan [3] if its curvature tensor R satisfies the relation $\nabla R=0$, where ∇ denotes the operator of covariant differentiation with respect to the metric tensor g.

During the past years, the notion of locally symmetric manifolds has been weakened by many authors in several ways to a different extent, semi$-$symmetric manifold by Szabo [28], pseudo-symmetric manifold in the sense of Deszcz [18], pseudo-symmetric manifold in the sense of Chaki [4]

A non-flat Riemannian manifold (M^n,g) $(n>2)$ is said to be pseudo-symmetric in the sense of Chaki [4] if it satisfies the following relation.

$$(\nabla_W R)(X,Y,Z,U)=2A(W)R(X,Y,Z,U)+A(X)R(W,Y,Z,U)$$
$$+A(Y)R(X,W,Z,U)+A(Z)R(X,Y,W,U)$$
$$+A(U)R(X,Y,Z,W) \tag{1}$$

i.e.,

$$(\nabla_W R)(X,Y)Z=2A(W)R(X,Y)Z+A(X)R(W,Y)Z+A(Y)R(X,W)Z$$
$$+A(Z)R(X,Y)W+g(R(X,Y)Z,W)\rho \tag{2}$$

for any vector field X,Y,Z,U and W, where R is the Riemannian curvature tensor of the manifold, A is a non-zero $1-$form such that $g(X,\rho)=A(X)$ for every vector field X. Such an $n-$ dimensional manifold is denoted by $(PS)n$.

Every recurrent manifold is pseudo-symmetric in the sense of Chaki [4] but not conversely. Also, the pseudo$-$symmetry in the sense of Chaki is not equivalent to that in the sense of Deszcz [17]. However, pseudo-symmetry by Chaki will be the pseudo$-$symmetry by Deszcz if and only if the non$-$zero $1-$form associated with $(PS)n$, is closed. Pseudo-symmetric manifolds in the sense Chaki have been studied by Chaki and Chaki [6], Chaki and De [7], De [9], De and Biswas [11], De, Murathan and Ozgür [14], Özen and Altay ([22],[23]), Tarafder ([31], [32]), Tarafder and De, [33] and others .

A Riemannian manifold is said to be Ricci symmetric if its Ricci tensor S of type $(0,2)$ satisfies $\nabla S=0$, where ∇ denotes the Riemannian connection. During the past years, the notion of Ricci symmetry has been weakened by many authors in several ways to a different extent such as Ricci semi$-$symmetric manifold [28], pseudo Ricci symmetric manifold by Deszcz [18], pseudo Ricci symmetric manifold by Chaki [5].

A non$-$flat Riemannian manifold (M^n,g) is said to be pseudo$-$Ricci symmetric [5] if its Ricci tensor S of type $(0,2)$ is not identically zero and satisfies the condition

$$(\nabla_X S)(Y,Z)=2A(X)S(Y,Z)+A(Y)S(X,Z)+A(Z)S(Y,X) \tag{3}$$

for any vector field $X,Y,Z,$ where A is a nowhere vanishing $1-$form and ∇ denotes the operator of covariant differentiation with respect to the metric tensor g. Such an $n-$ dimensional manifold is denoted by $(PRS)n$. The pseudo$-$Ricci symmetric manifolds have been also studied by Arslan et al. [1], Chaki and Saha [8], De and Mazumder [13], De, Murathan and Özgür [14], Özen [21], and many others.

The relation (3) can be written as

$$(\nabla_X Q)(Y)=2A(X)Q(Y)+A(Y)Q(X)+S(Y,X)\rho, \tag{4}$$

where ρ is the vector field associated to the $1-$form A such that $A(X)=g(X,\rho)$ and Q is the Ricci operator, i.e., $g(QX,Y)=S(X,Y)$ for all X,Y.

As a weaker version of local symmetry, the notion of locally $\phi-$symmetric Sasakian manifolds was introduced by Takahashi [30]. By generalizing the notion of locally $\phi-$symmetric Sasakian manifolds, De, Shaikh and Biswas [15] introduced the notion of $\phi-$recurrent Sasakian manifolds. In this connection De [10] introduced and studied $\phi-$symmetric Kenmotsu manifolds and in De [16] , Yildiz and Yaliniz introduced and studied $\phi-$recurrent Kenmotsu manifolds. Shaikh and Hui studied locally $\phi-$symmetric $\beta-$kenmotsu manifolds [25] and extended generalized $\phi-$recurrent $\beta-$Kenmotsu manifolds [26], respectively. Also, in [24] Prakasha studied concircularly $\phi-$recurrent Kenmotsu Manifolds. Recently, Shukla and Shukla [27] introduced and studied $\phi-$Ricci symmetric Kenmotsu manifolds.

The object of the present paper is to study $\phi-$pseudo symmetric and $\phi-$ pseudo Ricci symmetric Lorentzian $\beta-$Kenmotsu manifolds. The paper is organized as follows. Section 2 is concerned with preliminaries. Section 3 is devoted to the study of $\phi-$pseudo symmetric Lorentzian $\beta-$Kenmotsu manifolds and also $\phi-$pseudo concircularly symmetric Lorentzian $\beta-$ Kenmotsu manifolds. It is proved that every $\phi-$pseudo symmetric Lorentzian $\beta-$Kenmotsu manifold is an $\eta-$Einstein manifold. In section 4, we have studied $\phi-$pseudo Ricci symmetric symmetric Lorentzian $\beta-$ Kenmotsu manifolds.

II. PRELIMINARIES

A smooth manifold (M^n, g) $(n = 2m+1 \geq 3)$ is said to be an almost contact metric manifold [24] and [2] if it admits a $(1,1)$ tensor field ϕ, a vector field ξ, an $1-$form η and a Riemannian metric g which satisfy

$$\phi^2 X = X + \eta(X)\xi,, \tag{5}$$

$$\phi\xi = 0, \tag{6}$$

$$\eta(\phi X) = 0 \tag{7}$$

$$g(\phi X, Y) = g(X, \phi Y) \tag{8}$$

$$g(X, \xi) = \eta(X), \tag{9}$$

$$\eta(\xi) = -1 \tag{10}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y) \tag{11}$$

for all vector field X, Y on M^n.

An almost contact metric manifold $M^n(\phi, \xi, \eta, g)$ is said to be Lorentzian $\beta-$Kenmotsu manifold if the following condition hold [24].

$$\nabla_X \xi = \beta(X - \eta(X)\xi) \tag{12}$$

$$(\nabla_X \phi)Y = \beta\{g(\phi X, Y) - \eta(Y)\phi X\} \tag{13}$$

where ∇ denotes the Riemannian connection of g in a Lorentzian $\beta-$ kenmotsu manifold [24]

$$(\nabla_X \eta)Y = \beta\{g(X,Y) - \eta(X)\eta(Y)\} \tag{14}$$

$$R(X,Y)\xi = \beta^2\{\eta(X)Y - \eta(Y)X\} \tag{15}$$

$$R(\xi, X)Y = \beta^2\{\eta(Y)X - g(X,Y)\xi\} \tag{16}$$

$$\eta(R(X,Y)Z) = \beta^2\{\eta(Y)g(X,Z) - \eta(X)g(Y,Z)\} \tag{17}$$

$$S(X,\xi) = -(n-1)\beta^2\eta(X) \tag{18}$$

$$S(\xi,\xi) = -(n-1)\beta^2 \tag{19}$$

$$Q\xi = -(n-1)\beta^2\xi \tag{20}$$

$$S(\phi X, \phi Y) = S(X,Y) - (n-1)\eta(X)\eta(Y) \tag{21}$$

$$(\nabla_w R)(X,Y)\xi = \beta^2\{g(X,W)Y - g(Y,W)X - R(X,Y)W\} \tag{22}$$

for any vector field X,Y,Z on M^n and R is Riemannian curvature tensor and S is a Ricci tensor of type $(0,2)$, such that

$$g(QX,Y) = S(X,Y). \tag{23}$$

A Kenmotsu manifold is said to be $\eta-$Einstein if its Ricci tensors of type $(0,2)$ is of the form

$$S = ag + b\eta\otimes\eta \tag{24}$$

where a & b are smooth functions.

3 $\phi-$ PSEUDO SYMMETRIC LORENTZIAN $\beta-$ KENMOTSU MANIFOLDS

Definition 1 *A Kenmotsu manifold* $M^n(\phi,\xi,\eta,g)(n=2m+1)$ *is said to be* $\phi-$*pseudo symmetric Lorentzian* $\beta-$*Kenmotsu manifold if the curvature tensor* R *satisfies*

$$\begin{aligned}\phi^2((\nabla_w R)(X,Y)Z) &= 2A(W)R(X,Y)Z\\ &+A(X)R(W,Y)Z\\ &+A(Y)R(X,W)Z\\ &+A(Z)R(X,Y)W\\ &+g(R(X,Y)Z,W)\rho\end{aligned} \tag{25}$$

For any vector field $X,Y,Z,$ and W, where A is non zero $1-$form. If, in particular $A=0$ then the manifold is said to be $\phi-$symmetric $[10]$

we now consider a kenmotsu manifold $M^n(\phi,\xi,\eta,g)(n=2m+1)$ which is $\phi-$pseudo symmetric, then, by virtue of (5), it follows from (25) that

$$\begin{aligned}(\nabla_w R)(X,Y)Z+\eta((\nabla_w R)(X,Y)Z) &= 2A(W)R(X,Y)Z\\ &+A(X)R(W,Y)Z\\ &+A(Y)R(X,W)Z\\ &+A(Z)R(X,Y)W\\ &+g(R(X,Y)Z,W)\rho\end{aligned} \tag{26}$$

from which it follows that

$$\begin{aligned}g((\nabla_w R)(X,Y)Z,U) &= -\eta((\nabla_w R)(X,Y)Z)\eta(U)\\ &+2A(W)g(R(X,Y)Z,U)\\ &+A(X)g(R(W,Y)Z,U)\\ &+A(Y)(gR(X,W)Z,U)\\ &+A(X)g(R(W,Y)Z,U)\\ &+A(Y)g(R(X,W)Z,U)\end{aligned} \tag{27}$$

Taking an orthonormal frame field and then contracting (27) with respect to X and U and then using (8), (16),(22) and the relation

$$g\left(\left(\nabla_{w}R\right)\left(X,Y\right)Z,U\right)=g\left(\left(\nabla_{w}R\right)\left(X,Y\right)U,Z\right) \tag{28}$$

we have

$$g\left(\left(\nabla_{w}R\right)\left(\xi,Y\right)Z,\xi\right)=0 \tag{29}$$

by virtue of (29), it follows from (28) that

$$\begin{aligned}\left(\nabla_{w}S\right)\left(Y,Z\right)&=2A\left(W\right)S(Y,Z)+A\left(Y\right)S\left(W,Z\right)+A(Z)S\left(Y,W\right)\\&+A\left(R(W,Y)Z\right)+A\left(R\left(W,Z\right)Y\right)\end{aligned} \tag{30}$$

thus we can state that;

Theorem 1 *A $\phi-$pseudo symmetric Lorentzian $\beta-$Kenmotsu manifold is pseudo Ricci$-$symmetric if*

$$A\left(R(W,Y)Z\right)+A\left(R\left(W,Z\right)Y\right)=0$$

Setting $z=\xi$ in (25) and using (15), (17) and (22), we get

$$\begin{aligned}\left(1+A\left(\xi\right)\right)R\left(X,Y\right)W=\eta&\left(\left(\nabla_{w}R\right)\left(X,Y\right)\xi\right)\xi\\&+\beta^{2}\left\{g\left(X,W\right)Y-g\left(Y,W\right)X\right\}\\&-2A\left(W\right)\beta^{2}\left\{\eta\left(X\right)Y-\eta\left(Y\right)X\right\}\\&-A\left(X\right)\beta^{2}\left\{\eta\left(W\right)Y-\eta\left(Y\right)W\right\}\\&-A\left(Y\right)\beta^{2}\left\{\eta\left(X\right)W-\eta\left(W\right)X\right\}\\&-\beta^{2}\left\{\eta\left(X\right)g\left(Y,W\right)-\eta\left(Y\right)g\left(X,W\right)\right\}\rho\end{aligned} \tag{31}$$

Thus we can state that;

Theorem 2 *In a $\phi-$pseudo symmetric Sasakian manifold, the curvature tensor satisfies the relation* (31) *then we can get*

$$\begin{aligned}\left(1+A\left(\xi\right)\right)S\left(Y,W\right)=&\left\{1-n+A\left(\xi\right)\right\}\beta^{2}g\left(Y,W\right)\\&+2A\left(W\right)\beta^{2}\left\{n+1\right\}\eta\left(Y\right)\\&+A\left(Y\right)\beta^{2}\left\{n+1\right\}\eta\left(W\right)\\&+\beta^{2}\left(\eta\left(Y\right)\eta\left(W\right)\right)A\left(\xi\right)\end{aligned} \tag{32}$$

replacing Y by ϕY and W by ϕW in (32), we get

$$\left(1+A\left(\xi\right)\right)S\left(\phi Y,\phi W\right)=\left\{1-n+A\left(\xi\right)\right\}\beta^{2}g\left(\phi Y,\phi W\right) \tag{33}$$

by virtue of (11) and (21), we have (33)

$$S\left(X,Y\right)=a\,g\left(X,Y\right)+b\,\eta\left(X\right)\eta\left(Y\right) \tag{34}$$

where

$$a=\frac{\{1-n+A(\xi)\}\beta^{2}}{(1+A(\xi))}$$

and

$$b=\frac{\left[\{-n+1+A(\xi)\}\beta^{2}+(1+A(\xi))(n-1)\right]}{(1+A(\xi))}$$

provided

$$\left[\ 1+A(\xi)\neq 0\right]$$

Thus we can state that;

Theorem 3 *A $\phi-$ pseudo symmetric Lorentzian $\beta-$ Kenmotsu manifold is an $\eta-$ Einstein manifold.*

4 $\phi-$ PSEUDO CONCIRCULARLY SYMMETRIC LORENTZIAN $\beta-$ KENMOTSU MANIFOLDS

Definition 2 *A Lorentzian $\beta-$ Kenmotsu manifold $M^{n}(\phi,\xi,\eta,g)$ $(n=2m+1)$ is said to be $\phi-$ pseudo concircularly symmetric if the concircular curvature tensor C^{*} given by*

$$C^{*}(X,Y)Z=R(X,Y)Z-\frac{r}{n(n-1)}\{g(Y,Z)X-g(X,Z)Y\} \tag{35}$$

satisfies the relation

$$\begin{aligned}\phi^{2}\left((\nabla_{w}C^{*})(X,Y)Z\right)&=2A(W)C^{*}(X,Y)Z\\ &+A(X)C^{*}(W,Y)Z+A(Y)C^{*}(X,W)Z\\ &+A(Z)C^{*}(X,Y)W+g\left(C^{*}(X,Y)Z,W\right)\rho\end{aligned} \tag{36}$$

For any vector field X,Y,Z and W, where A is non zero $1-$ form and r is the scalar curvature of the manifold.

Let us consider a Kenmotsu manifold $M^{n}(\phi,\xi,\eta,g)(n=2m+1)$ which is $\phi-$ pseudo concircularly symmetric, then, by virtue of (5), it follows from (36) that

$$\begin{aligned}(\nabla_{w}C^{*})(X,Y)Z=&-\eta\left((\nabla_{w}C^{*})(X,Y)Z\right)\xi\\ &+2A(W)C^{*}(X,Y)Z+A(X)C^{*}(W,Y)Z\\ &+A(Y)C^{*}(X,W)Z+A(Z)C^{*}(X,Y)W\\ &+g\left(C^{*}(X,Y)Z,W\right)\rho\end{aligned} \tag{37}$$

from which it follows that

$$\begin{aligned}g\left((\nabla_{w}C^{*})(X,Y)Z,U\right)=&-\eta\left((\nabla_{w}C^{*})(X,Y)Z\right)\eta(U)\\ &+2A(W)g\left(C^{*}(X,Y)Z,U\right)\\ &+A(X)g\left(C^{*}(W,Y)Z,U\right)\\ &+A(Y)g\left(C^{*}(X,W)Z,U\right)\end{aligned} \tag{38}$$

$$+A(Z)g\left(C^{*}(X,Y)W,U\right)$$
$$+g\left(C^{*}(X,Y)Z,W\right)A(U)$$

Taking an orthonormal frame field and contracting (38) over X and U and then using (8), we get

$$\begin{aligned}(\nabla_{w}S)(Y,Z)=\frac{dr(W)}{n}g(Y,Z)-g\left\{(\nabla_{w}C^{*})(\xi,Y)Z,\xi\right\}\\+2A(W)S(Y,Z)+A(Y)S(W,Z)+A(Z)S(Y,W)\\-\frac{r}{n}\left[2A(W)g(Y,Z)+A(Y)g(W,Z)+A(Z)g(Y,W)\right]\\+A\left(C^{*}(W,Y)Z\right)+A\left(C^{*}(W,Z)Y\right)\end{aligned}\tag{39}$$

by virtue of (29), we have from (35) that

$$g\left\{(\nabla_{w}C^{*})(\xi,Y)Z,\xi\right\}=-\frac{dr(W)}{n(n-1)}\left\{g(Y,Z)-\eta(Z)\eta(Y)\right\}\tag{40}$$

In view of (40) it follows from (39) that

$$\begin{aligned}(\nabla_{w}S)(Y,Z)=-g\left\{(\nabla_{w}C^{*})(\xi,Y)Z,\xi\right\}\\+2A(W)S(Y,Z)+A(Y)S(W,Z)+A(Z)S(Y,W)\\-\frac{r}{n}\left[2A(W)g(Y,Z)+A(Y)g(W,Z)+A(Z)g(Y,W)\right]\\+\frac{dr(W)}{n(n-1)}\left\{ng(Y,Z)-\eta(Z)\eta(Y)\right\}\\+A\left(C^{*}(W,Y)Z\right)+A\left(C^{*}(W,Z)Y\right)\end{aligned}\tag{41}$$

thus we can state that;

Theorem 4 *A $\phi-$pseudo concircularly symmetric Lorentzian $\beta-$ kenmotsu manifold is pseudo Ricci symmetric if and only if*

$$\begin{aligned}\frac{r}{n}\left[2A(W)g(Y,Z)+A(Y)g(W,Z)+A(Z)g(Y,W)\right]\\-\frac{dr(W)}{n(n-1)}\left\{ng(Y,Z)-\eta(Z)\eta(Y)\right\}\\-A\left(C^{*}(W,Y)Z\right)-A\left(C^{*}(W,Z)Y\right)=0\end{aligned}\tag{42}$$

Setting $Z=\xi$ in (37) and using (15), (17), (22) and (35) we get

$$\begin{aligned}\left[\beta^{2}+A(\xi)\right]R(X,Y)W=\left[\beta^{2}-\frac{rA(\xi)}{n(n-1)}\right]\left\{g(X,W)Y-g(Y,W)X\right\}\\-\frac{dr(W)}{n(n-1)}\left\{\eta(Y)X-\eta(X)Y\right\}\end{aligned}\tag{43}$$

$$+\left[\frac{r}{n(n-1)}-\beta^{2}\right]\left[\begin{array}{c}\{\eta(Y)X-\eta(X)Y\}2A(W)\\+\{\eta(Y)W-\eta(W)Y\}A(X)\\+\{\eta(W)X-\eta(X)W\}A(Y)\\+\{\eta(Y)g(X,W)-\eta(X)g(Y,W)\}\end{array}\right]\rho$$

Thus we can state that;

Theorem 5 *In a* $\phi-$ *pseudo concircularly symmetric Lorentzian* $\beta-$ *kenmotsu manifold the curvature tensor satisfies the relation* (43)

hence we get

$$\left[\beta^{2}+A(\xi)\right]S(Y,W)=\left[\begin{array}{c}\beta^{2}\{(1-n)-(n+1)A(\xi)\}\\+\frac{2rA(\xi)}{(n-1)}\end{array}\right]g(Y,W)$$
$$-\frac{dr(W)}{n(n-1)}\{n+1\}\eta(Y)$$
$$+\left[\frac{r}{n(n-1)}-\beta^{2}\right]\left[\begin{array}{c}\{n+1\}\eta(Y)2A(W)\\+\{n+1\}\eta(W)A(Y)\end{array}\right] \tag{44}$$

replacing Y by ϕY and W by ϕW

$$\left[\beta^{2}+A(\xi)\right]S(\phi Y,\phi W)=\left[\beta^{2}\{(1-n)-(n+1)A(\xi)\}+\frac{2rA(\xi)}{(n-1)}\right]g(\phi Y,\phi W) \tag{45}$$

$$S(Y,W)=\frac{\left[\beta^{2}\{(1-n)-(n+1)A(\xi)\}+\frac{2rA(\xi)}{(n-1)}\right]}{\left[\beta^{2}+A(\xi)\right]}g(Y,W)$$
$$+\frac{\beta^{2}\{(1-n)-(n+1)A(\xi)\}}{\left[\beta^{2}+A(\xi)\right]}\eta(Y)\eta(W)$$
$$+\frac{\left[\frac{2rA(\xi)}{(n-1)}+\left[\beta^{2}+A(\xi)\right](n-1)\right]}{\left[\beta^{2}+A(\xi)\right]}\eta(Y)\eta(W) \tag{46}$$

$$\left[\beta^{2}+A(\xi)\neq 0\right]$$

by virtue of (11) and (21), we have from (46) that

$$S(Y,W)=\gamma g(Y,W)+\delta\eta(Y)\eta(W) \tag{47}$$

where

$$\gamma = \frac{\left[\beta^2\left\{(1-n)-(n+1)A(\xi)\right\}+\frac{2rA(\xi)}{(n-1)}\right]}{\left[\beta^2+A(\xi)\right]}$$

and

$$\delta = \frac{\left[\beta^2\left\{(1-n)-(n+1)A(\xi)\right\}+\frac{2rA(\xi)}{(n-1)}+\left[\beta^2+A(\xi)\right](n-1)\right]}{\left[\beta^2+A(\xi)\right]}$$

provided

$$\left[\beta^2+A(\xi)\neq 0\right]$$

Thus we can state that;

Theorem 6 *A $\phi-$pseudo concircularly symmetric Lorentzian $\beta-$Kenmotsu manifold is an $\eta-$Einstein manifold.*

5 $\phi-$ PSEUDO RICCI SYMMETRIC LORENTZIAN $\beta-$ KENMOTSU MANIFOLDS

Definition 3 *A Lorentzian $\beta-$ Kenmotsu manifold $M^n(\phi,\xi,\eta,g)(n=2m+1)$ is said to be $\phi-$pseudo Ricci$-$symmetric if the Ricci operator Q satisfies the relation*

$$\phi^2\left\{(\nabla_X Q)(Y)\right\}=2A(X)Q(Y)+A(Y)Q(X)+S(X,Y)\rho \tag{48}$$

for any vector field X,Y where A is non zero $1-$form if, in particular $A=0$ then the manifold is said to be $\phi-$symmetric

Let us consider a Kenmotsu manifold $M^n(\phi,\xi,\eta,g)$ $(n=2m+1)$ which is $\phi-$pseudo Ricci$-$symmetric. Then, by virtue of (5), it follows from (48) that

$$(\nabla_X Q)(Y)+\eta\left\{(\nabla_X Q)(Y)\right\}\xi=2A(X)Q(Y)+A(Y)Q(X)+S(X,Y)\rho \tag{49}$$

from which it follows that

$$\begin{aligned} &g\left\{\nabla_X Q(Y),Z\right\}-S(\nabla_X Y,Z)+\eta\left\{(\nabla_X Q)(Y)\right\}\eta(Z) \\ &=2A(X)S(Y,Z)+A(Y)S(X,Z)+S(X,Y)A(Z) \end{aligned} \tag{50}$$

putting $Y=\xi$ in (49) and using (12) and (18), we get

$$\left[\beta+A(\xi)\right]S(X,Z)=(n-1)\beta^2\left[2A(X)\eta(Z)+\eta(X)A(Z)-\beta g(X,Z)\right] \tag{51}$$

replacing X by ϕX and Z by ϕZ in (51) and using (5), we get

$$\left[\beta+A(\xi)\right]S(\phi X,\phi Z)=-(n-1)\beta^{3}\left[g(\phi X,\phi Z)\right] \tag{52}$$

In view of (11) and (21) we have from (52) that

$$S(X,Y)=\frac{-\beta^{3}(n-1)}{\left[\beta+A(\xi)\right]}g(X,Y)+\left\{\frac{-\beta^{3}(n-1)}{\left[\beta+A(\xi)\right]}+(n-1)\right\}\eta(X)\eta(Y) \tag{53}$$

$$\left[\beta+A(\xi)\neq 0\right]$$

which implies that the manifold under consideration is $\eta-$Einstein. Thus we can state the following;

Theorem 7 *Every $\phi-$pseudo Ricci$-$symmetric Lorentzian $\beta-$Kenmotsu manifold is an $\eta-$Einstein manifold.*

REFERENCES

[1] Arslan, K., Ezentas, R., Murathan, C., Özgür, C, On pseudo Ricci symmetric manifolds. Balkan J. Geom. and Appl., 6(2001), 1–5.

[2] Blair, D. E., Contact manifolds in Riemannian geometry. Lecture Notes in Math. 509, Springer-Verlag, 1976.

[3] Cartan, E., Sur une classe remarquable d'espaces de Riemannian. Bull. Soc. Math. France, 54(1926), 214–264.

[4] Chaki, M. C., On pseudo-symmetric manifolds. An. Sti. Ale Univ., " AL. I.CUZA" Din Iasi, 33(1987), 53–58.

[5] Chaki, M. C., On pseudo Ricci symmetric manifolds. Bulgarian J. Physics, 15(1988), 526–531.

[6] Chaki, M. C., Chaki, B., On pseudo−symmetric manifolds admitting a type of semi-symmetric connection. Soochow J. Math., 13(1)(1987), 1–7.

[7] Chaki, M. C., De, U. C., On pseudo-symmetric spaces. Acta Math. Hungarica, 54(1989), 185–190.

[8] Chaki, M. C., Saha, S. K., On pseudo-projective Ricci symmetric manifolds.Bulgarian J. Physics, 21(1994), 1–7.

[9] De, U. C., On semi-decomposable pseudo symmetric Riemannian spaces. Indian Acad. Math., Indore, 12(2)(1990), 149–152.

[10] De, U. C., On $\phi-$ symmetric Kenmotsu manifolds. Int. Electronic J. Geom., 1(1)(2008), 33–38.

[11] De, U. C., Biswas, H. A., On pseudo-conformally symmetric manifolds. Bull.Cal. Math. Soc., 85(1993), 479–486.

[12] De, U. C.. Guha, N., On pseudo symmetric manifold admitting a type of semi-symmetric connection, Bulletin Mathematique. 4(1992), 255–258.

[13] De, U. C., Mazumder, B. K., On pseudo Ricci symmetric spaces. Tensor N. S., 60(1998), 135–138.

[14] De, U. C., Murathan, C., Özgür, C., Pseudo symmetric and pseudo Ricci symmetric warped product manifolds. Commun. Korean Math. Soc., 25(2010), 615–621.

[15] De, U. C., Shaikh, A. A., Biswas, S., On $\phi-$ recurrent Sasakian manifolds. Novi Sad J. Math., 33(2003), 13–48.

[16] De, U. C., Yildiz, A., Yaliniz, A. F., On $\phi-$ recurrent Kenmotsu manifolds. TurkJ. Math., 33(2009), 17–25.

[17] Deszcz R., On pseudo-symmetric spaces. Bull. Soc. Math. Belg. Ser. A, 44(1)(1992), 1–34.

[18] Deszcz, R., On Ricci-pseudo-symmetric warped products. Demonstratio Math., 22(1989), 1053–1065.

[19] Kenmotsu, K., A class of almost contact Riemannian manifolds. Tohoku Math.J, $24(1972), 93-103$.

[20] Oubi ̈ na, J. A., New classes of almost contact metric structures. Publ. Math.Debrecen, $32(1985), 187-193$.

[21] Ozen, F., On pseudo $M-$projective Ricci symmetric manifolds. Int. J. Pure Appl.Math., $72(2011), 249-258$.

[22] Ozen, F., Altay, S., On weakly and pseudo symmetric Riemannian spaces. Indian J. Pure Appl. Math., $33(10)$ $(2001), 1477-1488$.

[23] Ozen, F., Altay, S., On weakly and pseudo concircular symmetric structures on a Riemannian manifold. Acta Univ. Palacki. Olomuc. Fac. rer. nat. Math., $47(2008), 129-138$.

[24] Prakasha,D.G.,Bagewadi,C.S. and Basavarajappa,N.S.;On Lorentzian $\beta-$Kenmotsu Manifolds int.JMA,Vol. $2, 919-927$

[25] Shaikh, A. A., Hui, S. K., On locally $\phi-$symmetric $\beta-$kenmotsu manifolds. Extracta Mathematicae, $24(3)(2009), 301-316$.

[26] Shaikh, A. A., Hui, S. K., On extended generalized $\phi-$recurrent $\beta-$Kenmotsu Manifolds. Publ. de l'Institut Math. (Beograd), $89(103)(2011), 77-88$.

[27] Shukla, S. S., Shukla, M. K., On $\phi-$Ricci symmetric Kenmotsu manifolds. NoviSAD J. Math., 39(2) (2009), 89–95.

[28] Szabo, Z. I., Structure theorems on Riemannian spaces satisfying $R(X,Y)R=0$, The local version. J. Diff. Geom., $17(1982), 531-582$.

[29] Tanno, S., The automorphism groups of almost contact Riemannian manifolds. Tohoku Math. J., 21 $(1969), 21-38$.

[30] Takahashi, T., Sasakian $\phi-$symmetric spaces. Tohoku Math. J., 29 $(1977), 91-113$.

[31] Tarafder, M., On pseudo symmetric and pseudo Ricci symmetric Sasakian manifolds. Periodica Math. Hungarica, 22 $(1991), 125-129$.

[32] Tarafder, M., On conformally flat pseudo symmetric manifolds. An. Sti. AleUniv., " AL. I. CUZA" Din Iasi, $41(1995), 237-242$.

[33] Tarafder, M., De, U. C., On pseudo symmetric and pseudo Ricci symmetric $K-$contact manifolds. Periodica Math. Hungarica, $31(1995), 21-25$.